Read This Guide First!

MATH S365 Graphs, networks and design

The Open University

MATH S365
Course Guide

SUP 00518 1

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MATH S365: start here

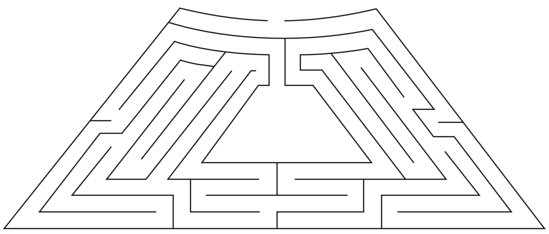
The purpose of this guide is to give you some idea of the content, philosophy and structure of the course, and to explain how we have designed the various components. We know that you will be keen to get started on the course units, but we suggest that you first spend about half an hour on this guide to get some general ideas about the course, and to avoid wasting time and effort later.

This course guide consists of three sections. In Section 1, *Introduction to* MATHS365, we list some typical problems to be solved in the course, and briefly describe the main themes: *combinatorics* and *mathematical modelling*. In Section 2, *Components of the course*, we discuss the structure of the correspondence texts, audio-CD, computer activities, assignments and the related topics.. Finally in Section 3, we outline *Content of the course*, we include a brief description of each unit. The back cover contains a list of units and a diagram showing how the units depend on each other.

1 Introduction to MATH S365

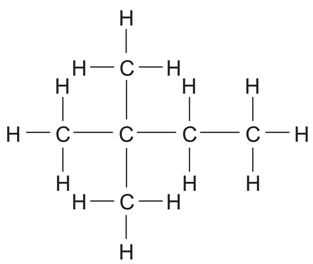
The following are just a few of the many problems you will look at in this course.

- Suppose you are stuck in the middle of a maze. Is there any guaranteed method for finding your way out?



Hampton Court maze

- When a spacecraft goes into orbit, it needs to be able to communicate with the Earth. How do you construct a suitable code to enable it to do so reliably?
- If you wanted to drive from Land's End to John O'Groats, how would you find the shortest route?
- If you try to colour a map of Europe in such a way that neighbouring countries are assigned different colours, you will find that only four colours are necessary. Is this true for all maps, or are there maps which need more colours?
- How many molecules are there with the formula C_6H_{14}?



a molecule

- Is it possible to tile a floor with a combination of regular 12-sided, 6-sided and 4-sided tiles?
- What is the most efficient way of bracing a given rectangular framework to make it rigid?
- How can you design a gas pipeline network at minimum cost, if there are restrictions on the amount of flow, pipe diameters, and cost per kilometre?

Although these problems may seem very diverse at first sight, they can all be expressed as problems involving the arrangements of certain objects and the relationships between these objects. By developing general methods for tackling problems of this kind, we shall show both how to solve such problems and also how to spot connections between problems which may appear at first sight to have little in common. By understanding the underlying reasons for these connections, you will gradually gain further insight into the nature of the original problems and their solutions.

The branch of mathematics which deals with such arrangement problems is known as *combinatorial analysis*, or **combinatorics**; the development of this subject is one of the themes of the course. We split the subject of combinatorics into three interrelated areas — **Graphs**, **Networks** and **Design**, each of which has four units devoted to it. We shall develop general methods for solving a variety of combinatorial problems, and then apply these methods to the above problems, and many others.

Many of the situations we examine in this course arise from important practical problems in technology and science. Indeed, much of the impetus given to the subject in the last few years has arisen out of the need to solve particular problems in industry involving network analysis or operational research. By representing these problems in combinatorial terms and applying combinatorial techniques of the kind discussed in this course, it has been possible to make substantial savings in cost and time. This brings us to another theme of the course — *mathematical modelling*.

In this course, the **mathematical modelling** process involves reformulating a problem in such a way that it can be approached by combinatorial techniques. This is not always easy since the way in which this modelling is carried out, and the degree to which the mathematical model faithfully represents the original problem, vary considerably from problem to problem. Throughout this course we emphasize not only the modelling process itself, but also its limitations.

So the two main themes of the course are the development of combinatorics as a subject in its own right, and the modelling of practical situations. The primary interest of the mathematician may well be the former, whereas for the technologist and scientist the problems themselves may well provide the main interest. We have tried to integrate these two approaches, since we believe that theory and practice are too interrelated to be separated successfully.

We hope that the course will prove of interest to everyone — whether technologist, scientist or mathematician. For the mathematician the course will provide an opportunity to see mathematics in action in solving worthwhile problems, and for the technologist and scientist the course will show the importance and usefulness of developing a mathematical framework which can be used to interrelate different problems, and provide means for solving them.

The subject you are about to embark on is an exciting one. Although its roots go back a long way, it is a modern subject in which substantial advances are being made all the time. It is likely to play an ever-increasing role in the years to come, and this course is designed to give you the necessary background to understand these future developments.

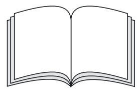
2 Components of the course

MATHS365 is a 10-credits course, and consists of fourteen units. Each unit is intended to take approximately 10–12 hours of study time, and consists of a correspondence text and some assignment questions. Several units also involve computer activities, and a related audio-tape sequences, and other supplementary material. There is no set book, summer school or home kit, and there are no radio programmes. There is a three-hour examination at the end of the course.

Academic Timetable

With the first mailing you should receive **a Academic Timetable** for the course. You should study this carefully, as it will tell you which units have associated audio sequences, and the cut-off date of an assignment, and tutorials.

Correspondence texts



Most units have five sections, each of which is designed to be approximately an evening's work. Each section ends with a list of objectives that provide a useful checklist to which you should refer after reading the section.

Study guide and Introduction

Each correspondence text begins with a *Study guide*, followed by an *Introduction*.

The *Study guide* includes a *Plan of the unit* — a diagram giving the titles of the various sections, and indicating how these sections depend on one another; audio-CD and computer symbols are included in the appropriate places in the *Plan* and also in the text. The *Study guide* also gives guidance on how to plan your study of the unit. In some cases you may wish to study the sections in a different order from that suggested (for example, if you are working on a bus or train), and the *Plan* and *Study guide* will indicate whether this is possible.

The *Introduction* gives a brief overview of the unit, and outlines the content of each section.

Problems

You will be working through many problems while studying the texts, so you will need a pencil and paper handy. The solutions to the problems are provided at the back of the unit. You will gain most benefit if you attempt each problem before looking at our solution. You should certainly read each solution before proceeding, even if you are confident that your solution is correct, as our solutions sometimes contain additional comments or material relevant to the subsequent text, and indicate whether other solutions are possible.

Exercises

Each unit contains a set of exercises, similar to the problems, which you can work through to gain extra practice, or use for revision purposes. The solutions are given in the section following the exercises.

Further reading

Near the end of each unit is a list of relevant further reading material, in case you wish to study some of the topics in greater depth.

Computing

Most units involve computer activities, for which you will need your home computer, the course software and the *Computer Activities* booklet.

Computer

To run the course software, your computer needs to conform to the specification for this course, details of which are given in the *Computer Activities* booklet.

Do not worry if you are not familiar with computers. The assumptions we make about your ability to use your computer are minimal, and are described in the *Computer Activities* booklet. This booklet also guides you through some of the course software, to give you a feeling for how it works before you embark on the computer activities for the units.

Do not hesitate to contact the Academic Computing Service (ACS) Help Desk if you have a problem with your computer. The Help Desk staff would prefer you to contact them with a small problem as it occurs, rather than that you spend hours struggling, as this may lead you to lose confidence in yourself and the course materials. The staff are experienced PC users. The Help Desk hours and contact numbers will be published in a Stop Press.

It is important to read and work through the *Computer Activities* booklet before starting the *Introduction* unit.

Software

The course software can be downloaded from MATH S365 OLE under the folder **<Computing Software/setup>**.
You save the setup.exe into your own computer and then install it.

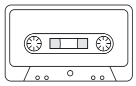
Computer activities

Most of the computer activities are designed to help you consolidate or develop ideas that occur in the units. The activities are described in the *Computer Activities* booklet and are best attempted at the points in your study indicated in the units. However, you should not find yourself seriously disadvantaged if you need to delay your attempts at these activities until you have completed the related unit.

We are confident that you will find the activities useful and fun. *However, it is very easy to spend too much time on the computer work*. You should be aware of this danger and ration your computing time accordingly. We have indicated in many cases the maximum amount of time you should spend on an activity. We have also indicated when an activity is optional. Although the optional activities are often fun, don’t attempt them if you are short of time. Remember, if you wish to spend extra time playing, do not blame the Course Team if you get behind with your work on the course!

Audio-CDs

Several units (usually the Networks unit s) have an audio-CDs sequence associated with certain sections. You will need to play relevant sequence while working on these sections. While listening to each audio-CD sequence, you will need to follow the relevant frames in the given **Audio Notes 1 or 2.**



The audio-tapes have been replaced by audio CDs but the contents are unchanged.

Sometimes you will need to stop the **CD** to tackle a problem or answer a question, so you should make sure that the 'pause' or 'stop' button is within easy reach while you are working, and that you have a pencil and paper handy. The points at which you should stop the **CD** are indicated by an audible signal, but you may wish to stop the CD in other places in order to replay it or to take a break.

Handbook

There is a course Handbook which summarizes the main results of each unit. You will not be allowed to bring the MATH S365 Course Handbook to the exam. Another copy of the MATH S365 Handbook will be given to you together with the exam paper.

Important: Carefully read the Handbook Regulation.

Assessment

The course is assessed by means of four written assignments and two assignments in the form of multiple-choice questions - these will be referred to as 'assignments' and assignments (MC) in the texts below for short. Also there is a three-hour examination at the end of the year. Each assignment and assignment (MC) is associated with three or four units, as indicated in the list of units at the end of this *Course Guide*. The continuous assessment and final examination account for 30% and 70% of the course score. Further details about the method of awarding a course credit are given in *Student Handbook.*

The cut-off dates for the assignments and assignments (MC) are given in the *Study Calendar*. You should submit your assignments to arrive on or before the corresponding cut-off dates. An assignment (MC) which arrives after the cut-off date will not be marked under any circumstances. Ideally, you should send in each assignment to arrive about a week before the cut-off date: then if you have left out a page, for example, you can be contacted in time. Note that it may be better to send in a partially completed assignment than to fall behind on the course.

Some of the assignments and assignments (MC) questions *require* you to use the course software. For other questions, *unless otherwise stated in the question*, you may use the course software to help you answer the question if you wish.

Assignment

Before completing your first assignment, read the following notes carefully.

- All the questions in each assignment are assessed.
- When submitting solutions, remember to fill in your name, personal identifier and page number on each answer sheet.
- Use A4 paper, leaving enough room for your tutor's comments.
- The number of marks assigned to each part of a question is printed in the *Assignment Booklet*, and should give you an indication of the relative difficulty or importance of each question. Do not write several pages on a question assigned 1 mark, or two lines for a question assigned 10 marks!
- Always read the question carefully. To help you, we adopt the following conventions on wording:

  'write down' and 'state' mean 'write down without justification';

  'find', 'determine', 'calculate', 'construct' and 'evaluate' mean that we require you to show all your working ;

  'prove' and 'show' mean 'justify each step'.

  If you use a definition, result or theorem to get from one line to the next, you should quote its number, or the page of the *Handbook* or unit on which it appears. When using a theorem, always demonstrate that the conditions of the theorem are satisfied.
- For questions *requiring* the use the course software, you should submit as part of your answer appropriate *annotated* printout as specified in the question. For other questions permitting the use of the course software, you may submit printout to supplement your answers if you wish.
- If you have difficulty with any question, seek help and advice from your tutor.
- Tutors are asked to comment fully on each question and to use every opportunity to teach on the basis of your solutions. Always try to write something down for each question, as your tutor will not know how to help you if you just leave a blank space. Writing 'I'm stuck here' or 'I need to be able to prove so-and-so, but I can't' will help your tutor to help you.
- If you have difficulty in meeting a cut-off date, contact your tutor as early as possible.
- When you get the assignment back, spend some time studying your tutor's comments, and rewrite any wrong answers along the lines your tutor suggests. You will find such time well invested when you come to use the assignments for revision purposes. We regard these comments by your tutor as a major part of the teaching of the course.

Assignment (MC)

Before completing your first assignment (MC), read the following notes carefully.

- All the questions in each assignment (MC) are assessed.
- Always mark your assignment (MC) form clearly, as otherwise you may lose marks unnecessarily.
- In this course there are no penalty marks for wrong answers. It is therefore better to make an intelligent guess than to leave a blank space.
- Answers for Assignments (MC) should be submitted through the 'OLE Assignment (multiple choice)' submission component. If you cannot access the Internet, please contact the course coordinator or the Assignment Office to make special arrangement for the submission of your answers.

Final examination

The three-hour final examination consists of two parts. **Part 1** consists of routine short-answer questions, designed to test your coverage of the course. **Part 2** consists of longer questions, with a certain amount of choice, designed to test a deeper understanding of the course material.

There are many topics covered in the course, and you are not expected to remember them all, so we have produced a *Handbook* (see above) together with the examination paper.

A *Specimen Examination Paper* and *Solutions* will be sent to you during the year. These will give you an idea of the format of the examination, and the types of question which will be set. You should study the solutions carefully so that you know what sort of answer you will need to write down in order to get full marks.

Stop presses

These contain important up-to-date information on various aspects of the course, particularly on the computing. You should read and act upon these as soon as you receive them.

Workload

Your personal circumstances may not permit you to study every part of every unit. If this happens, you should concentrate on those parts of the unit which are covered by the assignments. If necessary, you should ration the time you spend on the computer activities. It is more important to keep up with the course than to understand every single detail of each unit before proceeding to the next!

Comments on the course

We shall be pleased to receive your comments on MATHS365. Please send comments, constructive criticisms, and details of any mistakes you discover to:

Course Coordinator: Dr. Tony Chan

Email: tmtchan@ouhk.edu.hk
Office Telephone: 2768 6867

Online Learning Environment (OLE)

OLE is a Web-based learning system (Internet system) developed by the OUHK and is an interactive online learning environment used for communication among students, tutors and the Course Coordinator.

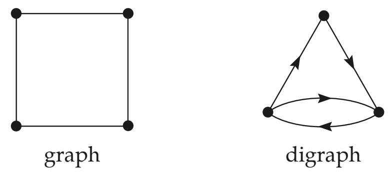
This system can enhance student's learning experience through its interactive tools. To help you to use the OLE system, refer to the User Guide: "Online Learning Environment". MATH S365 students are required to access the OLE for the submission of assignments (MC).

3 Content of the course

The course comprises an *Introduction* unit, four units on each of the topics *Graphs*, *Networks* and *Design*, and a *Conclusion* unit. We sometimes refer to a particular trio of units such as *Graphs 1*, *Networks 1* and *Design 1* as 'Block 1'. The three main areas are as follows.

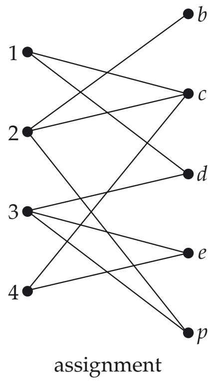
Graphs

In this course the word 'graph' usually refers to a diagram of points interconnected by lines, rather than to a picture representing a function. The points may correspond to towns on a map, atoms in a molecule, people, and so on; the lines connecting the points may correspond to roads between the towns, bonds between the atoms or friendships between the people. Digraphs (short for 'directed graphs') are like graphs, except that each line has a direction indicated by an arrow. A digraph may be used, for example, to represent a one-way street system.



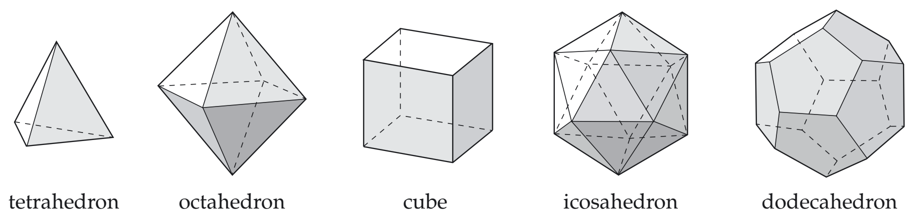
Networks

How can you find the shortest route between two towns on a road map? How can a manufacturer send the required amount of a commodity to a number of markets at minimum cost? How do you make the best assignment of a number of candidates to a number of jobs? These are examples of the types of network problem you will meet in this course.



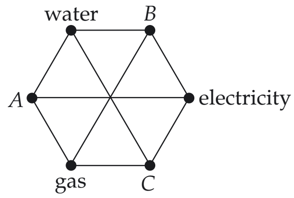
Design

These units cover four main topics: *geometric design*, *kinematic design*, *design of codes* and *block designs*. The examples considered include plane tilings, the design of a robot manipulator or an aeroplane simulator, the design of error-correcting codes such as those that have been used in interplanetary space probes, and the design of experiments.



Outlines of the units

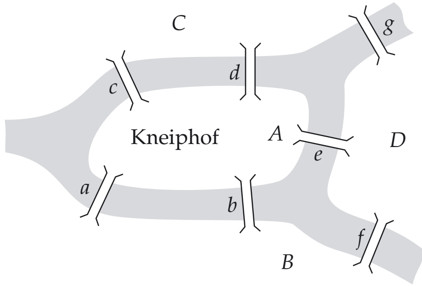
Introduction



houses and utilities

What is combinatorics, and what can it do for you? This unit introduces the subject, and discusses the nature of algorithms and mathematical modelling. Examples are taken from the three main areas: *Graphs*, *Networks* and *Design*. This unit also introduces the computer activities by inviting you to investigate particular problems. General methods for solving these problems are given in later units.

Graphs 1 Graphs and digraphs

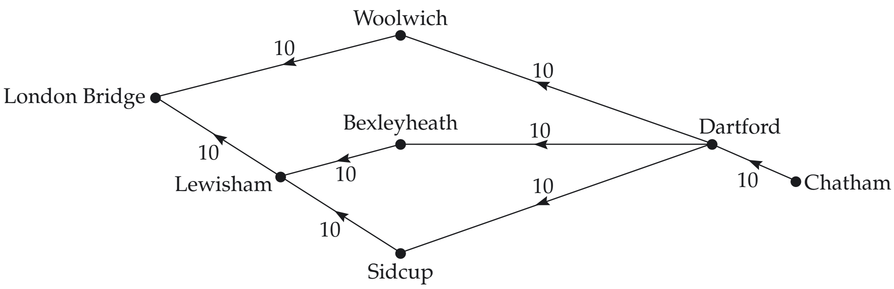


Königsberg bridges

Here we discuss the properties of graphs and digraphs in general, and describe a number of applications. The applications include the use of graphs in chemistry, genetics, ecology and music, and the use of digraphs in the social sciences. We also discuss Eulerian and Hamiltonian graphs, and related problems; one of these is the well-known Königsberg bridges problem.

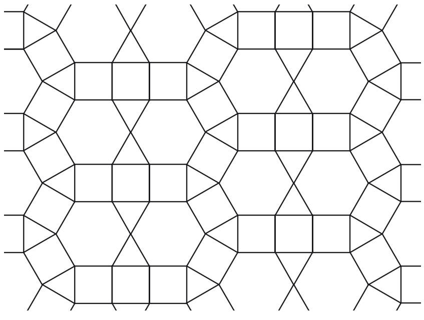
Networks 1 Network flows

We are concerned here with the problem of finding the maximum amount of a commodity (gas, oil, passengers) which can pass between two points of a network in a given time. We give an algorithm for solving this problem, and discuss a number of important variations which frequently arise in practice.



Design 1 Geometric design

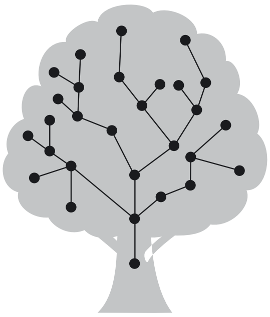
This unit is mainly concerned with the shape and/or arrangement of objects. In particular, it discusses tilings in two dimensions, and regular and semi-regular solids in three dimensions. We discuss the idea of the dimension of a space or an object, and the concept of convexity. The unit ends with the more abstract concept of an incidence structure, examples of which you will meet in *Design 3* and *Design 4*.



tiling

Graphs 2 Trees

Trees are graphs which occur in areas such as branching processes, decision procedures, the representation of molecules, and linguistics. After discussing the mathematical properties of trees, we look at various applications, such as the minimum spanning tree problem, its use in obtaining an approximate solution to the travelling salesman problem, and to a multi-terminal flow problem in which a commodity can flow between each pair of vertices in a network.



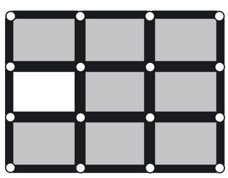
tree

Networks 2 Optimal paths

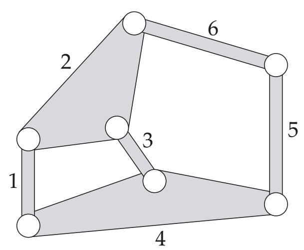
How does an engineering manager plan a complex project involving many activities? This is one of a class of problems in which the shortest or longest paths in a graph or digraph are sought. In particular, this unit looks at problems involving scheduling activities for complex industrial processes, and shows how a 'critical path' method can be used to tackle some of these problems.

Design 2 Kinematic design

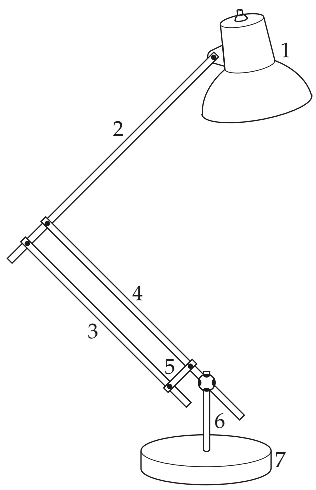
The mechanical design of desk lamps, robot manipulators, aeroplane simulators and space-frame structures, together with a variety of other artefacts, depends on the interconnection of systems of rigid bodies. This unit discusses the contribution of combinatorial ideas to this area of engineering design. The concepts involved have practical importance for the technologist as well as being of theoretical interest to the mathematician.



Is this a rigid structure?



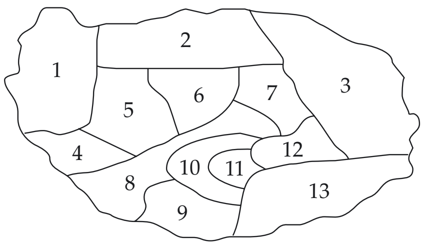
a kinematic system



links in a desk lamp

Graphs 3 Planarity and colouring

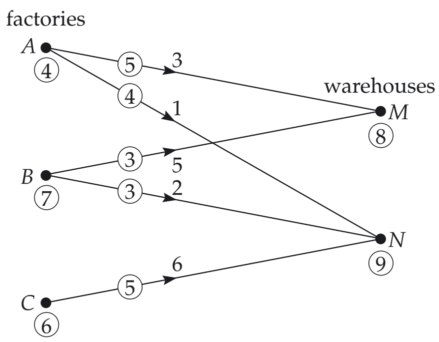
When can a graph be drawn in the plane without its lines crossing? Is it possible to colour the countries of any map with just four colours in such a way that neighbouring countries are coloured differently? These are two of a number of apparently unrelated problems considered in this unit, which have applications to the design of printed circuits and the scheduling of refuse-collection lorries.



map

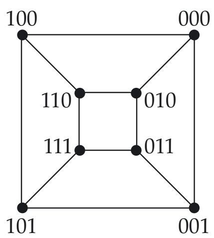
Networks 3 Assignment and transportation

If there are ten applicants for ten jobs, and each applicant is suitable for only a few of the jobs, under what conditions is it possible to fill all the jobs? And if the applicants are ranked according to their suitability for each job, what is the best way of assigning them to jobs? If a manufacturer wishes to supply a number of warehouses with some product made in a number of factories, how can the warehouses be supplied at minimum cost? These are some of the systems-management type of problem considered in this unit.

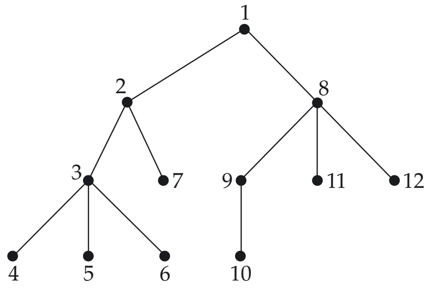


Design 3 Design of codes

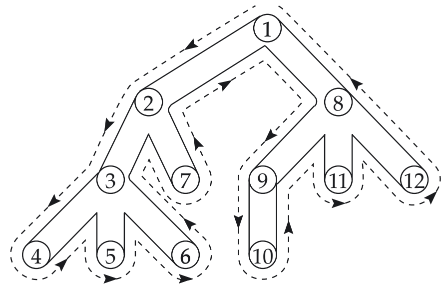
In a communication system, imperfect reception can sometimes be overcome by the inclusion of redundant information. In formal terms, we can code information before transmission so that a receiver can both detect and correct errors. This unit discusses the properties of particular codes which arise in practice — for example, cyclic codes and Hamming codes, and some codes used in space probes.



Graphs 4 Graphs and computing



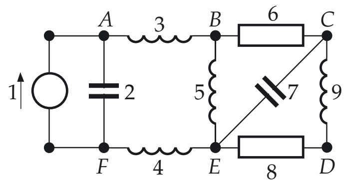
labelled tree



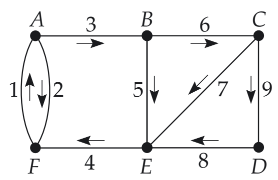
wall diagram

This unit describes some important uses of graphs in computer science. Among the topics discussed are the complexity of algorithms, sorting algorithms, tree-searching methods such as depth-first and breadth-first searches, and the 'knapsack problem'. The knapsack problem is that of choosing from a number of items, each having a given value and weight, those having a maximum total value subject to a fixed total weight limit.

Networks 4 Physical networks



electrical network

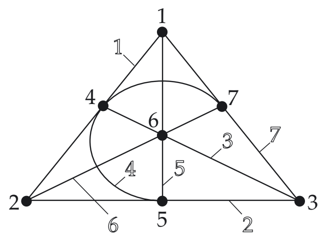


graphical representation

Graph theory and network theory provide an appropriate structured language for representing systems of combinatorial complexity. Sometimes the mathematical structure predicts things that must happen, or things that can never happen. This unit shows how the mathematics studied so far can be applied, and then extended, to give a more general theory of the connectivity which constrains the dynamic behaviour of physical and social systems.

Design 4 Block designs

If an agricultural research station wants to test a number of varieties of a crop, how can this be done in a fair and balanced way? The answer to this type of problem lies in the study of block designs. This unit explains the concepts of balanced and resolvable designs, and describes methods for constructing block designs with these properties.



Fano plane

Conclusion

The aim of this final unit is to bring together some of the themes of the course: combinatorics, mathematical modelling and algorithms, and to remind you of some of the ways in which they have appeared in the course. The unit summarizes and brings together many of these ideas.

4. Course Aims and Learning Outcomes

Aims

1. Introduce the three interrelated areas of combinatorics, i.e. graphs, networks and design, as well as the application of combinatorial techniques and mathematical modelling, to solve practical problems presented by current technology, operational research, business and the decision sciences.

2. Provide an opportunity to see discrete mathematics in action in solving worthwhile problems.

Learning Outcomes

At the end of the course, you should be able to:

1. Discuss a wide variety of real-life problems and their solutions in three interrelated areas: graphs, networks and design;

2. Analyse, interpret and evaluate different types of discrete mathematical models to solve different real-life applications;

3. Demonstrate the importance and usefulness of developing a mathematical framework which can be used to interrelate different problems, and provide a means for solving them;

4. Analyse and evaluate solutions using computing software;

5. Demonstrate professional skill and intellectual ability in applying decision science techniques to various applications.

Course team

The course was produced by the team:

Joan Aldous (Academic Editor, Author)

Steve Best (Graphic Artist)

Keith Cavanagh (Publishing Editor)

Penny Day (Course Manager)

Alan Dolan (Reader)

Anne Marie Gallen (BBC Producer)

Adam Gawronski (Software Designer)

Jennifer Harding (Publishing Editor)

Fred Holroyd (Author)

John Jaworski (BBC Producer)

Jeff Johnson (Author)

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Tim Lister (Author)

Roger Lowry (Publishing Editor, Author)

Rob Lyon (Graphic Designer)

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Roy Nelson (TV)

Joe Rooney (Author, TV)

Jon Rosewell (Software Designer)

David Saunders (BBC Producer)

Richard Scott (Software)

Robin Wilson (Chair, Author)

The external assessor was:

Lowell Beineke (Professor of Mathematics, Indiana University–Purdue University)

Valuable assistance was given by:

Mathematics and Computing Faculty Course Materials Production Unit

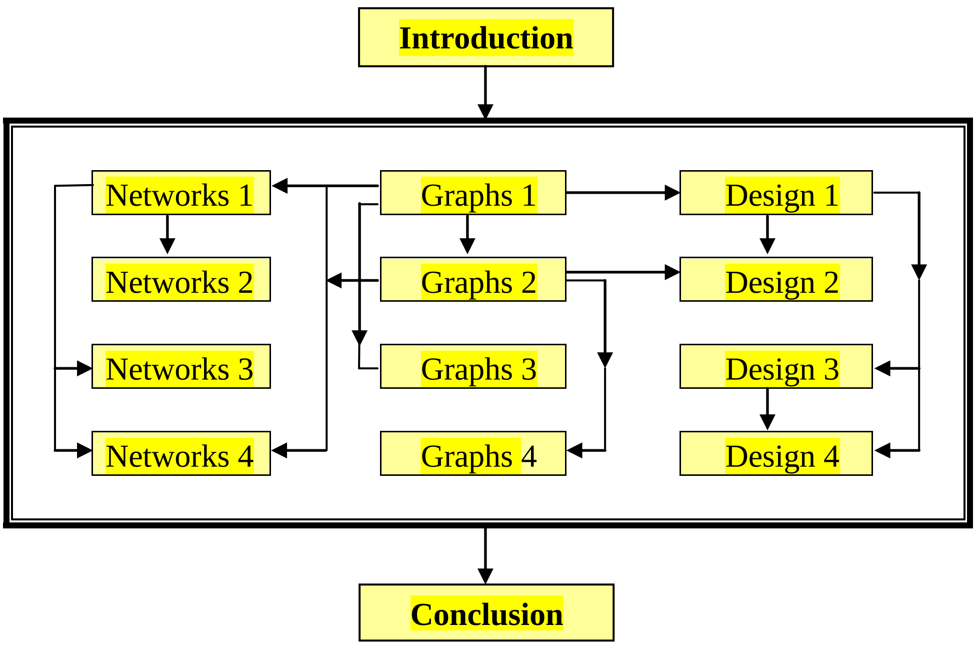
Educational Software Group, ACS

David Asche (Software), Ian Every (Software), Noel Eastham (Author), Caroline Hailey (Graphic Designer), Chris Rowley (Software), Pip Surgey (Audio), Howard Taylor (Graphic Artist), Helen Thompson (Software), Mike Williams (Course Manager)

Assessment Coverage and Unit Relationship

	Units	Title	Assessment coverage	
	Introduction			
Block 1	Graphs 1	Graphs and Digraphs	Assignment 01	Assignment (MC) 41
	Networks 1	Network Flows		
	Design 1	Geometric Design		
Block 2	Graphs 2	Trees	Assignment 02	
	Networks 2	Optimal Paths		
	Design 2	Kinematics Design		
Block 3	Graphs 3	Planarity and Colouring	Assignment 03	Assignment (MC) 42
	Networks 3	Assignment and Transportation		
	Design 3	Design and Codes		
Block 4	Graphs 4	Graphs and Computing	Assignment 04	
	Networks 4	Physical Networks		
	Design 4	Block Designs		
	Conclusion			

Inter-relationship between Units



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