

MATH S390

**Quantitative Models for
Financial Risk**

Course Guide



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Production

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1. Introduction

Welcome to *MATH S390 Quantitative Models for Financial Risk*. We hope that you will find your study of this course enjoyable and rewarding.

This *Course Guide* contains information about the content and components of *MATH S390 Quantitative Models for Financial Risk* that you should be aware of before starting to study the course. Like most HKMU courses, *MATH S390* combines the study of written materials with other activities.

2. What this course is about

This course will help you to develop professional skills in using quantitative models, as well as their applications for analysis of financial risk and for critically evaluating the statistical and physical properties of Brownian motion in pricing stock options.

MATH S390 starts with the basic principles and assumptions for options — the concept of no arbitrage — and teaches you the valuation of European and American options through the binomial model and the Black-Scholes option pricing model. After completing the course, you will be able to apply the stochastic and continuous-time models to compute the pricing of financial options and other derivative securities for the assessment of risk.

2.1 Prerequisites

You are advised to have studied a basic statistics course such as *MATH S280* and a mathematics course such as *MATH S221*, or to have achieved an equivalent level of mathematical maturity before studying this course.

2.2 Course aims

MATH S390 Quantitative Models for Financial Risk aims to:

- provide you with a basic understanding of various option pricing formulas, hedging techniques, bond models and interest rates;
- introduce different types of financial risks and the common quantitative methods used to set up financial derivative models, and to analyse and interpret various problems and risks arising in financial engineering;
- introduce basic statistical and mathematical theory, in particular the stochastic and continuous-time differentiation models required for computing the pricing of financial options and other derivative securities for the assessment of risk;
- enhance your ability to elaborate on the assumptions for developing options models, and to equip you with the ideas of forwards and options and the concept of non-arbitrage in order to consider the pricing of financial derivatives;
- equip you to use the binomial model and the Black-Scholes option pricing model to interpret the valuation of European and American options;
- develop your professional skills in stochastic calculus and its applications for risk analysis and finance;

- teach you the properties of Brownian motion and how to apply them to evaluate the price of stock options problems; and
- develop quantitative financial risk models through the multi-variable calculus and differential equations.

2.3 Course learning outcomes

Upon completing *MATH S390*, you should be able to:

- *Define* and *explain* basic terminology in the financial risk and quantitative process.
- *Determine* the option value involving one or more sets of cash flows at specified times, and *apply* the binomial tree model to price options.
- *Apply* the stochastic calculus for modelling Brownian motion, and *use* Ito's Lemma to derive the Stochastic Differential Equations.
- *Apply* the non-arbitrage principle and risk neutral (martingale) pricing.
- *Derive* a partial differential equation for the Black-Scholes option pricing model, *solve* the Black-Scholes differential equation, and *interpret* the computational results.
- *Construct* a hedge for a variety of risk positions, *derive* and *apply* the Delta Hedging Equation, and *develop* numerical methods for pricing financial products and hedging strategies for a portfolio of options.
- *Perform* calculations of yields, continuous compounding, and par, forward and zero yield curves.

2.4 Course organization

Units		Weeks	Assessment
1	<i>Introduction to financial risk and quantitative process</i>	2	
2	<i>Tree models for stocks and options</i>	3	Assignment 1
3	<i>Mathematical methods for the Black-Scholes model</i>	2	
4	<i>Risk models in hedging</i>	2	Assignment 2
5	<i>Quantitative methods for bond models and interest rate options</i>	3	
6	<i>Financial risk models in practice</i>	3	Assignment 3
Revision		1	

3. Components of the course

A typical fortnight's work will consist mainly of reading a course unit and working through the exercises that it contains.

Each unit has continuous assessment associated with it. There are three assignments, of which the best two will count towards your final score. The course ends with a two-hour examination.

3.1 Course units

There are six course units. Each unit is divided into four to seven sections. A typical section might be studied in a single session (in an evening, for example). Each unit begins with an introductory section that asks you to recall what you learned in a prerequisite course or from previous units as you begin to study the current one, and also gives advice on which sections may be most or least time-consuming. Towards the end of the unit, you will find a short Summary section that reminds you of the operations you should be able to do as a result of having studied the unit. This will help ensure that you have mastered the contents of one unit before moving on to the next.

Examples and self-tests

The course units contain various types of questions for you to work through as part of your study.

First, there are examples, which show you how to carry out some techniques or methods. The solution to an example is given in the text immediately below it; your task is to read and to follow the workings of the problem in order to learn how to apply that technique yourself.

Next, self-tests are designed to give you practice in achieving what the preceding text has taught you. You should attempt to solve these by yourself, consulting the solutions (which are given at the back of the unit) only in order to check that your own answer is correct. If you are stuck, look at the solution as a last resort, but look at just enough of it to see how to proceed, before returning to complete the rest of the solution of the problem for yourself. It cannot be emphasized too strongly that doing self-tests in this way is an essential part of studying mathematics; nobody learns much mathematics just by reading texts.

You should try each example within a section as you come to it. At the end of most sections you will find additional self-tests that provide extra practice if you need it; these self-tests may be a little more demanding than the majority of the exercises within the sections. You may regard these self-test exercises as optional, but it is recommended that you do as many of them as you can find the time for.

3.2 MATH S390 Handbook

This is designed as a work of reference, and provides a convenient source of basic definitions and formulas for use throughout the course. In addition, you will be given a handbook in the examination room.

The Handbook has two main components: a collection of useful formulas and definitions, many of which you will have come across already in prerequisite courses, and summaries of the main concepts, definitions and techniques in each of the course units. The Handbook also summarizes particular formulas from the course which need to be called upon regularly, for rapid reference.

It is a good idea to start using the Handbook right from the beginning of the course so that you become familiar with its contents.

3.3 Stop press notices

The stop presses act as a course newsletter containing useful and often essential information such as errata and details of tutorial arrangements. It is important that you read each stop press as soon as you receive it. These notices are also posted in the Online Learning Environment (OLE).

3.4 The Online Learning Environment (OLE)

The Online Learning Environment User Guide (<http://ole.hkmu.edu.hk/help.html>) explains to you the hardware and software requirements for you to access the course electronically. It also helps you to use the components in the OLE. Through the OLE, you can get more information on the course and communicate with other students and tutors of the course.

3.5 Academic Timetable

This gives the starting date for each unit, the dates when assignments are due and weekends when tutorials are scheduled.

3.6 Tutorials

Dates for tutorials are given in the Presentation Schedule. Other details, such as tutorial venues and exact timing, will be given to you through emails and the OLE. Attend tutorials to meet your tutor and the other students on the course. Be active in sharing your views in tutorials.

3.7 Assessment

The course has both a continuous assessment and examination assessment component.

The distribution of marks on these assessments towards the overall course score is set out in the following table.

Assessment type	Weighting of the course score
Assignments	30%
Final examination	70%
Total	100%

You will be awarded the full five credits for *MATH S390* if you can get at least 40 marks on both the OCAS and the examination. Read the *Student Handbook* for information on the awarding of course results.

Assignments

There are three assignments for the course, of which the best two results will count towards your final score. Since these assignments are important for you to secure the concepts you have learned in the study units, you will be required to submit all three of the assignments. Upon receiving assignments from the students, tutors will be required to mark the assignments and return them to students with their comments and feedback.

The assignments will require you to:

- perform the derivation of a continuous model;
- apply a theorem and solve the given model;
- analyse case studies in order to value a option; and
- complete a computer project using Excel.

The marks for the best two assignments will be distributed as follows:

	Coverage	Weighting of the assignments
Assignment 1	This assignment covers <i>Units 1–2</i> . There will be 3–4 problem-solving exercises.	15%
Assignment 2	This assignment covers <i>Units 3–4</i> . There will be 3–4 problem-solving exercises.	+
Assignment 3	This assignment covers <i>Units 4–6</i> . There will be 3–4 problem-solving exercises.	15%
		(best 2 of 3 assignments)

Examination

The two-hour final examination for *MATH S390* will be ‘closed book’, with the exception of the course Handbook. The examination is worth 70% of the total marks for the course. The exam paper will be divided into two parts:

- Part I will contain some short questions that assess your general knowledge of the course material from all units.
- Part II will comprise more challenging long questions based on a problem-solving approach. The questions will assess your skills in quantitative modelling; in applying certain models to measure the financial risk problems; and in concluding results for recommendation.

Questions in assignments and in the examination carry both *accuracy marks and method marks*. You should therefore, as a general practice, show all your work to solve each problem.

We expect you to leave numbers like π and $\sqrt{2}$ as they are, but you should simplify expressions such as $\sin(\pi/2)$. If you need to use decimal fractions at any time, two decimal places will normally be sufficient.

3.8 Calculators and mathematics software

You can use a calculator, mathematics software such as Scientific Notebook, or spreadsheet software such as Microsoft Excel to evaluate mathematical functions such as exponential, logarithmic, trigonometric (and their inverses) and hyperbolic (and their inverses) functions when you study the course.

Calculators are allowed in the examination. The University has a List of Approved Models of Calculators so that students realize what types of calculators are allowed in the examination. This List is updated according to the types of calculators approved by the Hong Kong Examinations and Assessment Authority. You will receive the List from Registry before the examination.

You are not allowed to use a non-approved calculator or a calculator without the ‘HKEA/HKEAA Approved’ label in the examination. For your early information, a copy of the approved calculator list is attached at the back of this guide.

4. Developer profiles

Professor James Caldwell obtained his BSc(Hons) and MSc Degrees from the Queen's University of Belfast in 1964 and 1966, respectively. In 1974 he obtained his PhD in Applied Mathematics from Teesside University. He took up various teaching posts in the UK before moving to Australia as Head of Mathematics at the University of Southern Queensland. He then returned to teach at Sunderland University and worked in lecturing and research posts at a number of UK universities. In 1990 he joined the City University of Hong Kong as Adjunct Professor in the Department of Mathematics.

Professor Caldwell was awarded his first higher doctorate (DSc) from Queen's University of Belfast in 1985, and his second DSc from Teesside University in 2007 in recognition of his scholarly work in Mathematical Modelling.

In parallel with his academic career, Professor Caldwell worked for a number of large organizations, including as Head of Modelling for Unilever Research UK. Through his research work, he has published hundreds of articles and conference papers, and more than a dozen textbooks and theses. As a result of scientific publications, he has had extensive experience in editorial work involving Mathematics. Furthermore, he has had extensive experience in course development work in Mathematics at a number of universities.

Dr Douglas Kei-shing Ng received his BSc (1st class Hons) and MPhil degrees in Applied Mathematics from the City University of Hong Kong in 2001 and 2003, respectively. Between 2003 and 2006, he was a teacher in a secondary school. He joined the Hong Kong Polytechnic University in 2006 to undertake his PhD research on medical informatics. He has been a part-time tutor of Mathematics at Hong Kong Metropolitan University since 2006 and is now a full-time Lecturer for HKMU. His research interests include Nonlinear Partial Differential Equations, Mathematical Modelling, Computer Aided Detection Methods for Cerebrovascular Diseases, and Meshless Computational Methods.

Dr Chi-wang Chan obtained his BSc (1st class Hons) and MPhil degrees in Physics from the Chinese University of Hong Kong in 2000 and 2003, respectively. From 2002 to 2003, he was a Physics and Mathematics subject teacher in a secondary school. From 2003 to 2009, he went to New York University for his PhD studies in Statistical Physics, with specialization in Transports and Brownian motion in Confined Systems. He has been a part-time tutor of Mathematics at Hong Kong Metropolitan University since 2010. His academic interests are in the fields of Stochastic Process, Quantitative Finance, Astrophysics and Science education.

List of approved calculators for examinations

An updated list will be sent to you before the examination.

In addition to the following models, calculators bearing the 'HKEA/HKEAA Approved' labels are also allowed.

A.MAX

SC-801	SC-802	SC-809	SC-813
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ATABA/AURORA

AC-688	AC-689	AC-690	AC-692
AC-693	AC-694	AT-1	AT-105
AT-106 A	AT-108 A	AT-168	AT-208 N/B
AT-231 A/B/C/D	AT-232 /S	AT-233	AT-241 T
AT-244 H	AT-256 H	AT-268	AT-281 /S
AT-282	AT-283	AT-368	AT-508
AT-510	AT-512	AT-518	AT-520
AT-522	AT-601 A	AT-620 A	AT-630
AT-687	AT-2129 A/B	AT-6120	AT-6320
AT-9300	BD-1	BD-2	D-8 /N
D-10 /N	D-12 N	SC-170	SC-180
SC-200	SC-500		

BISTEC

B300	B500	B600	B700
B900			

BLT

BT-206	BT-2016-12	BT-2018-12	DC-308-8S/12
DC-318-8S/12	DC-338-8S/12	DC-408	DC-508
SC-183			

CANON

BS-100	BS-102	BS-120	BS 122
BS-123	BS-200	BS-300	BS-1200TS
CB II BK/G	CB III	F-45	F-65
F-73 /P	F-402	F-500	F-502
F-600	F-602	F-604	F-612
F-700	F-800 P	F-802 P	FC-4 S
FC-42 S	FINANCIAL/II	FS-400	FS-600
HS-20H	HS-100	HS-102H	HS-120L
HS-1200RS/T/TV/TS	KC-20	KS-10	KS-20
KS-30	KS-80	KS-100	KS-102

KS-120	KS-122	KS-123	L-20 II W AD
L-30 II W AD	L-813 II	L-1011	L-1214II/AD
L-1218	LC-22	LC-23	LC-34 /T
LC-44	LC-63	LC-64 T	LC-101
LC-500H	LC-1016	LC-1222	LC-1620H
LS-8	LS-21	LS-25H II	LS-31 II
LS-32	LS-39H	LS-41 II	LS-42
LS-43 B/S	LS-51	LS-52 BK/W	LS-54 W
LS-61	LS-62 BK/W	LS-80/H	LS-81 Z
LS-82 H/Z	LS-88Hi/V	LS-100 II/H/TS	LS-102 Z
LS-120H/L/RS/V	LS-151	LS-500	LS-510
LS-550 G/B1	LS-552	LS-553	LS-560
LS-562	LS-563	LS-566H	LS-716H
LS-1000H	LS-1200H	M-10	M-20
M-30	OS-1200	PS-8 BK/W	PS-10BK/W
SK-100H	T-14BK/G/W	T-19	TR-10H
TR-1200H	TS-81/H	TS-83	TS-85H
TS-101	TS-103	TS-105H	TS-120TL
TX-1210Hi	WS-100	WS-120	WS-121H
WS-200H	WS-220H	WS-1200H	WS-1210Hi
WS-2222	WS-2224	WS-2226	

CASIO

AZ-45F	BF-80	BF-100	CV-700
D-20A	D-20D/M	D-40D	D-100 W/L/LA
D-120	DF-10L	DF-20L	DF-120TE
L/W/T/LA/TE			
DJ-120	DN-10	DN-20	DN-40
DS-1 B/L	DS-2 B/L	DS-3/L	DS-8 E
DS-10E/L/G	DS-20 E/L/G	DS-120	FC-100
FN-10	FN-20	FX-8	FX-10 F
FX-39	FX-50 F	FX-55	FX-61 F
FX-68 /B	FX-78	FX-82/B/C/D/L/ LB/SUPER/SX/W	FX-85 /M/N/V
FX-100/A/B/C/V/D	FX-115 /M/N/V/D	FX-120	FX-135
FX-140	FX-210	FX-350/A/C/D/H/ HA/W	FX-451 M
FX-500 /A	FX-550 /S	FX-570 A-/C-/V/ D/S	FX-911S/SA
FX-991/M/N/V/D/ H/S	FX-992 V/VB/S	FX-3400 P	FX-3600 P/V/A/ PV
FX-3650P	FX-3800 P	FX-3900PV	FX-3950P
HL-100 L	HL-122/L	HL-812 /E/L	HL-820 A/LU/D
HS-4A	HS-8 G/L/LU/D	HS-9	HS-88

HS-90	J-10 A/D	J-20	J-30 C
J-100W/L/LA	J-120 L/W/T	JE-2	JE-3
JF-100/TE	JF-120TE	JL-210	JN-10
JN-20	JN-40	JS-8 C	JS-10 /C/M/L/LA
JS-20/C/M/L/LA	JS-25	JS-40 L/LA	JS-110
JS-120	JS-140	LC-401A	LC-403 C/E/L/ LU/LB
LC-700	LC-710	LC-787 G/GU	LC-797 G/GU
LC-798 G	LC-1000 /L	LC-1210	MC-40 S
MC-801 S	MJ-20	MJ-120	MS-5A
MS-6	MS-7/LA	MS-8 W/A	MS-9
MS-10 W/L	MS-20W/TE	MS-70 L	MS-100 A/TE/V
MS-120 A/TE/V	MS-140 A	MS-170 L/LA	MS180
MS-270 L/LA	MS-470 L	NS-3	NS-10L
NS-20L	RC-770	S-1	S-2
S-20 L	SJ-20	SL-80 E	SL-100 A/B
SL-110 A/B	SL-120 A/B	SL-200	SL-210
SL-220	SL-240/L	SL-300H/J/L/LH/ LU/LB	SL-310 M
SL-330	SL-350	SL-450	SL-510 /A
SL-704	SL-720 /L	SL-760 A/C/LU/ LB	SL-787
SL-790L	SL-797	SL-805A	SL-807 /A/L/LU
SL-817 L	SL-850	SL-910L	SL-1000 M
SL-1200L	SL-1510	SL-1530T	SL-2000 M
US-20	US-100	WD-100L	WD-120L
WJ-10	WJ-20	WJ-100L	WJ120L

CITIZEN

CT-500	CT-600	ELS-301	ELS-302
ELS-501	F-908 /N	F-920	F-940 N
F-950	FT-200	LC-505	LC-508 N
LC-510 N	LC-516 N	LC-531	LC-5001
LH-700	LH-830	SB-741 P	SDC-810
SDC-814	SDC-826	SDC-830	SDC-833
SDC-834	SDC-836	SDC-839	SDC-848
SDC-850	SDC-865	SDC-868	SDC-875
SDC-878	SDC-880	SDC-888	SDC-8001
SDC-8150	SDC-8360	SDC-8401	SDC-8460
SDC-8480	SDC-8780/L	SDC-8890	SLD-702
SLD-705 B	SLD-707	SLD-708	SLD-711 /N
SLD-712 /N	SLD-720	SLD-722	SLD-723
SLD-725	SLD-732	SLD-735	SLD-737

SLD-740	SLD-742	SLD-750	SLD-760
SLD-767	SLD-781	SLD-7001	SLD-7401
SR-30	SR-35	SR-70	SR-260
SRP-40	SRP-45	SRP-60	SRP-65
SRP-75	SRP-80	SRP-285II	

HEWLETT-PACKARD

HP-6S	HP-6S Solar	HP-9S	HP-10 B/BII
HP-11 C	HP-12 C	HP-15 C	HP-16 C
HP-20 S	HP-21 S	HP-30S	

KARCE

KC-107	KC-117	KC-119	KC121
KC127	KC-153	KC159	

SHARP

EL-231C/L	EL-233G	EL-240C	EL-310A
EL-326L/S	EL-330A	EL331A	EL-334H/A
EL-337M	EL338A	EL-344G	EL-354L
EL-373	EL376G	EL386L	EL387L
EL-480G	EL-501V	EL-506A/G/R/V	EL-509G/D/S/L/R/ V
EL-520 D/G/L/R/ V	EL-530 A	EL-531 GH/H/P/ LH/RH/VH	EL-546D/G/L
EL-556G/L	EL-731	EL-733A	EL-771C
EL-782C	EL-792C	EL-879L	EL-2125
EL-2128H	EL-2135	EL-5020	

TEXAS INSTRUMENTS

BA-III	BA-SOLAR	BA-II/PLUS	BA-35
BUSINESS-EDGE	FINANCIAL- INVESTMENT- ANALYST	MATH EXPLORER	TI-COLLEGIATE
TI-25X SOLAR	TI-30 /Xa/Xa Solar/XIIB	TI-31	TI-32
TI-34 /II TI-65	TI-35 /X	TI-36 /X Solar	TI-60

TRULY

101 /A	102	103	105
106	107	P-127	SC-106A
SC-107B/C/F	SC-108	SC-109 /X	SC-110 /X
SC-111 /X	SC-118 /A/B	SC-128	

[End of calculator list]